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EVOLUTION OF THE SHAPE OF THE ANODE BOUNDARY UNDER ELECTROCHEMICAL DIMENSIONAL MACHINING OF METALS

L. M. Kotlyar and N. M. Minazetdinov

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This paper presents a method for calculating the anode boundary under unsteady conditions of electrochemical dimensional machining of metals. The plane quasistationary problem of determining the shape of the anode boundary for various machining times is considered.

Key words: *electrochemical machining of metals, shape of the anode boundary, boundary element method.*

Introduction. In the electrochemical machining process, one can distinguish an initial unsteady stage of machining [1]. In this case, the distribution of the metal dissolution velocity over the machined surface and the local interelectrode distances vary in time. The configuration of the machined surface varies, tending to a certain asymptotic shape similar to the shape of the tool (cathode). In the present paper, we propose a mathematical model and a method for calculating the anode boundary in the initial stage of machining.

Model of the Process. The plane problem of electrochemical dimensional machining of metals under unsteady conditions is considered. Rectangular coordinates x_1 and y_1 are attached to the cathode (Fig. 1). It is assumed that the cathode moves in the negative direction of the ordinate.

The description of shape changes of a machined surface under unsteady conditions involves an evolutionary problem with a moving boundary and a nonstationary distribution of parameters. This problem is solved using a method in which the solution is found sequentially in particular time intervals reckoned from the initial specified state. The problem is formulated within the framework of an ideal process model. The main assumptions of the model and their detailed substantiation are given in [1]. According to this model, in the case of a direct current, the electric field in the interelectrode gap can be considered potential; i.e., $\boldsymbol{E} = -\operatorname{grad} u$, where $\boldsymbol{E}$ is the electric-field intensity vector and u is the electric field potential. In the ideal process, the electric field can be described by the Laplace equation $\nabla^2 u = 0$. The potentials u_{a} and u_{c} on the anode and cathode surfaces (Γ_{a} and Γ_{c}, respectively) are constant [1].

The linear velocity V_{a} of anode dissolution along the normal to the anode surface is given by the formula (see [1])

$$V_{\mathrm{a}} = \eta(i_{\mathrm{a}})\varepsilon i_{\mathrm{a}}/\rho, \tag{1}$$

where $i_{\mathrm{a}} = æ\partial u/\partial n_{\mathrm{a}}$ is the anode current density, $æ$ is the specific electric conductivity of the medium, ε is the electrochemical equivalent of the metal, ρ is the density of the anode material, and $\boldsymbol{n}_{\mathrm{a}}$ is the outward normal vector to the anode (Fig. 1). The current efficiency η depends on various process parameters, mainly on the anionic composition and concentration of the electrolyte, the chemical composition and hardness of the metal, and the current density. Condition (1) takes into account that η is a function of the current density.

The curves of current efficiency versus anode current density for machining of 5KhNM steel in NaCl and $NaNO_3$ solutions of various concentration obtained in the experiments of [2] are presented in [3]. By approximation of the experimental data and using expression (1), we obtain

$$V_{\mathrm{a}} = \frac{\varepsilon}{\rho}\,(a_1 + a_0 i_{\mathrm{a}}) = \frac{\varepsilon}{\rho}\Big(a_1 + a_0 æ\,\frac{\partial u}{\partial n_{\mathrm{a}}}\Big),$$

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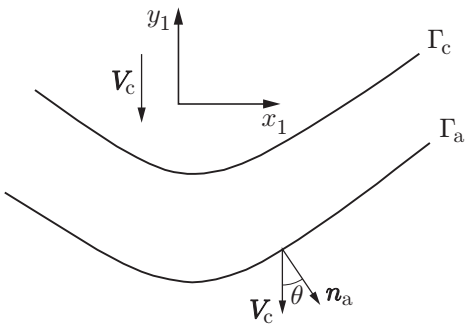

Fig. 1. Coordinate system of the surface boundaries of the anode and cathode.

where a_0 and a_1 are the constant approximation coefficients for the curve of current efficiency versus anode current density [3].

We introduce the characteristic current density $i_0 = \rho V_c/\varepsilon$ (V_c is the feed velocity of the cathode) and the characteristic length $H = æ(u_a - u_c)/i_0$ [3] and convert to the dimensionless variable

$$\psi = (u - u_c)/(u_a - u_c), \qquad x = x_1/H, \quad y = y_1/H, \quad n = n_a/H.$$

Then,

$$V_a = \frac{\varepsilon}{\rho}\, a_1 + \frac{\varepsilon æ(u_a - u_c)}{\rho H}\, \frac{\partial \psi}{\partial n} = \frac{\varepsilon}{\rho}\, a_1 + a_0 V_c\, \frac{\partial \psi}{\partial n} = V_c\Big(\frac{a_1}{i_0} + a_0\, \frac{\partial \psi}{\partial n}\Big). \tag{2}$$

The function ψ corresponding to the electric-field potential satisfies the Laplace equation in the interelectrode gap:

$$\nabla^2 \psi = 0.$$

On the boundaries of the electrodes, the following conditions are satisfied:

$$\psi_a = 1, \qquad \psi_c = 0.$$

On the electrically insulated surfaces, the following condition is satisfied:

$$\frac{\partial \psi}{\partial n} = 0.$$

It is known that in the machining process, it is possible to distinguish unsteady and steady-state stages [1]. In the latter case, metal dissolution occurs with a velocity distribution that ensures parallel motion of points of the anode at a velocity equal to the cathode feed velocity. Under unsteady conditions,

$$V_a = V_c \cos\theta, \tag{3}$$

where θ is the angle between the cathode feed velocity $\boldsymbol{V}_c$ and the $\boldsymbol{n}_a$ normal vector to the anode (see Fig. 1). Using (2) and (3), we find that at points of the steady-stated anode boundary, the following condition should be satisfied:

$$\frac{a_1}{i_0} + a_0\, \frac{\partial \psi}{\partial n} = \cos\theta. \tag{4}$$

In the formulation and solution of problems of electrochemical dimensional machining of metals, it is common to use a hydrodynamic analogy of an electric field, according to which a plane potential field is replaced by a dummy flow of an ideal incompressible fluid. The hydrodynamic analogy facilitates the formulation of boundary-value problems in theory and allows the use of computational methods developed in solving hydrodynamic problem [1].

Formulation of the Problem. We consider the plane problem of electrochemical machining using a cathode tool consisting of straight-line segments and a semicircular cylindrical lobe of radius R (Fig. 2). The vector $\boldsymbol{V}_c$ indicates the direction of cathode feed. The initial position of the anode boundary Γ_1 is specified arbitrarily. We assume that at the initial time, the anode boundary is a straight line parallel to the abscissa. In the numerical

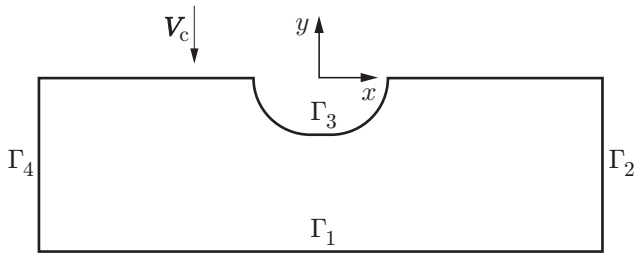

Fig. 2

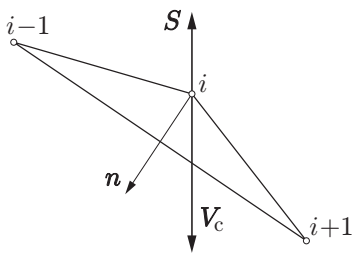

Fig. 3

Fig. 2. Computed geometry of the device.

Fig. 3. Diagram of computation nodes.

calculation, the dummy inflow and outflow are cut off at right angles to the initial velocity direction at certain distances from the lobe. The cutoff lines correspond to the inflow (Γ_2) and outflow (Γ_4) regions.

The problem reduces to solving the Laplace equation for the dimensionless potential ψ of the electric fields in the region bounded by the cathode boundary Γ_3, the cutoff lines Γ_2 and Γ_4, and the anode boundary Γ_1.

On the boundaries of the region, the function ψ satisfies the following conditions: $\psi = 1$ on the boundary Γ_1, $\psi = 0$ on the boundary Γ_3, and $\partial\psi/\partial n = 0$ on the cutoff lines Γ_2 and Γ_4. The latter condition indicates the absence of a velocity component normal to the principal direction of the dummy flow.

During the solution of the problem, it is required to determine the position of the anode boundary Γ_1 at various times until condition (4) is satisfied on the anode boundary.

Algorithm of Finding the Anode Boundary. The problem is solved using the boundary element method [4] with a linear variation in the functions ψ and $\partial\psi/\partial n$ on the element. During the calculations, it is necessary to check that at points of the anode boundary the value of $\Delta = |a_1/i_0 + a_0\,\partial\psi/\partial n - \cos\theta|$ decreases at each subsequent time. Otherwise, as shown by calculations, there is instability of the solution.

The problem is solved by the following scheme.

1. The anode boundary is considered known for the kth time.

2. The boundary-value problem is solved, and discrete values of $(\partial\psi/\partial n)_i$ are determined at the nodes of the anode boundary.

3. The displacements of points of the anode are determined as the sum of the vectors of displacements due to dissolution ($\Delta\boldsymbol{n}_a = \boldsymbol{V}_a\Delta t$) and cathode feed ($\Delta\boldsymbol{S} = -\boldsymbol{V}_c\Delta t$); here Δt is the time interval. In this case, a problem arises to represent the normals to the corner points with adequate accuracy. In this study, according to the scheme given in Fig. 3, it is assumed that the normal to the ith nodal point is perpendicular to the segment connecting the nodes adjacent to the ith nodal point. If we introduce the dimensionless time $\tau = V_c t/H$, the projections of the displacement of the nodal points of the anode boundary on the x and y axes, according to (2), can be defined by the following difference formulas:

$$\Delta x_i = \frac{y_{i+1} - y_{i-1}}{l_i}\Big(\frac{a_1}{i_0} + a_0\Big(\frac{\partial\psi}{\partial n}\Big)_i\Big)\Delta\tau, \qquad \Delta y_i = \Big(1 - \frac{x_{i+1} - x_{i-1}}{l_i}\Big(\frac{a_1}{i_0} + a_0\Big(\frac{\partial\psi}{\partial n}\Big)_i\Big)\Big)\Delta\tau.$$

Here $l_i = \sqrt{(x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2}$, x_i and y_i ($i = \overline{2, n-1}$) are the node coordinates and n is the number of nodes on the anode boundary.

4. The position of the anode boundary for the following $(k+1)$th time is defined by the formulas

$$x_i^{(k+1)} = x_i^{(k)} + \Delta x_i, \qquad y_i^{(k+1)} = y_i^{(k)} + \Delta y_i.$$

The process is then continued for the next time.

In the numerical solution of the problem for the $(k+1)$th time, the value of $\Delta_{k+1} = \max\limits_i |a_1/i_0 + a_0(\partial\psi/\partial n)_i - (\cos\theta)_i|$ at the nodes of the anode boundary is calculated. If $\Delta_{k+1} \geqslant \Delta_k$, the value of $\Delta\tau$ decreases by a factor of two and the coordinates of points of the anode boundary are recalculated with a new step $\Delta\tau$.

Results of Numerical Experiments. The calculations were performed for the following conditions. The radius of the cylindrical lobe was $R = 1.0$, the initial position of the anode corresponded to the straight line $y = -h$ ($h > 0$), the characteristic current density was $i_0 = 100$ A/cm^2, and the coefficients $a_0 = 0.906$ and $a_1 = -12.817$

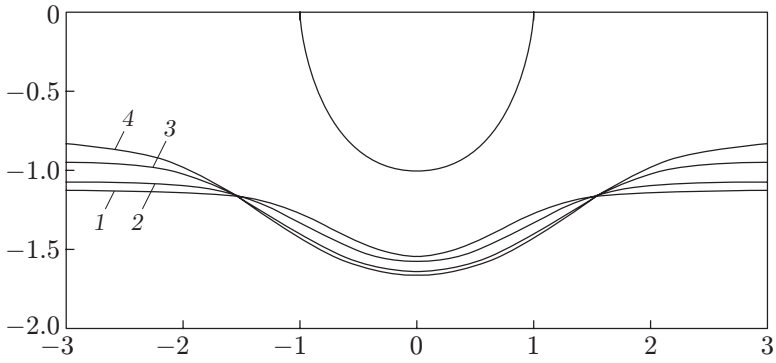

Fig. 4. Calculated anode boundaries at $h = 1.2$ and $\tau = 0.4062$ (1), 0.5938 (2), 1.2188 (3), and 3.4188 (4).

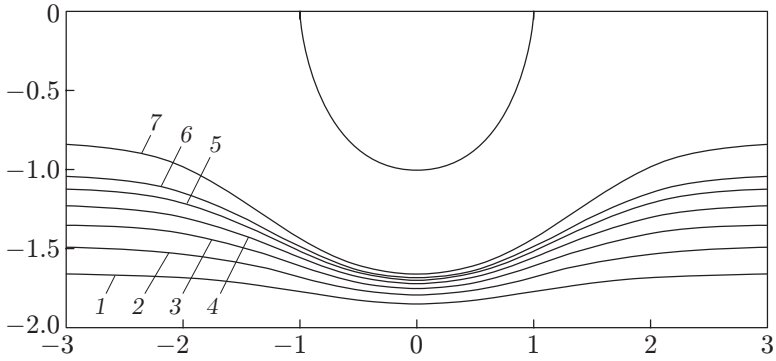

Fig. 5. Calculated anode boundaries at $h = 2.0$ and $\tau = 0.7375$ (1), 1.0500 (2), 1.3625 (3), 1.6750 (4), 1.9877 (5), 2.300 (6), and 4.8033 (7).

corresponded to 5KhNM steel in a 15% solution of $NaNO_3$. The boundary of the region was discretized using linear elements.

Numerical implementation of the method involves at least three problems: the choice of the sections Γ_2 and Γ_4, the choice of a step in time, and the choice of a criterion for the termination of the calculation.

To determine the effect of the choice of the boundaries of the sections Γ_2 and Γ_4, we performed calculations of the points of the anode boundary for the same times and different lengths of the interelectrode gap along the x axis equal to 10, 12, and 16. The calculation results for the common segments of the indicated gaps almost coincided; therefore, in the further study, the calculation results were considered for a gap of $[-6; 6]$. In this case, the boundary of the region was divided into 150 linear elements, and the anode boundary into 75 elements. The nodal points immediately under the cylindrical lobe were made finer to increase the calculation accuracy.

The step in time was selected so as to increase the accuracy of satisfaction of relation (4) for the next time. If this condition was not satisfied, the step in time was decreased.

In the calculations for the specified conditions, it was assumed that the initial step in time $\Delta\tau = 0.1$. After the first nine steps, the step in time, being sequentially decreased, became equal to 0.00625. This value of the step in time was not changed until condition (4) was satisfied on the anode boundary with a certain accuracy ε; after that, the value of $\Delta\tau$ decreased rapidly and almost vanished and the shape of the anode boundary ceased to change.

From the aforesaid it is clear that for the termination of the calculation, one can require satisfaction of condition (4) with required accuracy ε or to specify the minimum value of $\Delta\tau$ upon reaching which the shape of the anode boundary practically does not change.

Figures 4 and 5 show the calculation results for two initial positions of the anode at $h = 1.2$ and 2.0, respectively.

The anode boundaries corresponding to the steady-state conditions are shown in Fig. 4 (curve 4) and Fig. 5 (curve 7). The value of $\Delta = |a_1/i_0 + a_0\,\partial\psi/\partial n - \cos\theta|$ varies along the anode boundary from 0.0003 at a node located on the symmetry axis to 0.0091 in the neighborhood of the sections Γ_2 and Γ_4.

The results obtained for the steady-state anode boundary coincide with the calculation results for the stationary anode boundary given in [3].

Thus, in this study, we implemented the two-dimensional mathematical model proposed in [1, 3] for the initial stage of the ideal process of electrochemical dimensional machining of metals. Anode-boundary shapes for various machining times were obtained for the same cathode configuration. The calculations results show that the anode boundary takes a steady-state shape with time.

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